APPENDIX

The company's liabilities are composed of the following two parts: equity and debt. The equity does not receive dividends and the debt is in the form of a zero coupon bond with face value equal to D and maturity at time T.

If at time T, the value of the of the assets, A, is greater than the value of the debt, the company will pay its debt. If at time T, the value of the of the assets, A, is smaller than the value of the debt, the company will go bankrupt. Bondholders will receive the value of the assets and the shareholders will not receive anything. The company cannot go bankrupt before time T.

Formalizing this description: the value of the assets is assumed to follow a geometric Brownian motion described by the following equation:

$$dA = \mu_A \, A \, dt + \sigma_A \, A \, dW$$

where,
μ_A is the drift of the asset value - assumed to be equal to zero in this case;
σ_A is the volatility of the company's assets;
dW is a standard Wiener process.

The value of the assets at time t is then equal to

$$A_t = A_0 exp\left\{\left(\mu_A - \frac{\sigma_A^2}{2}\right)t + \sigma_A^2 \sqrt{t}\, W_t\right\}$$

where $W_t \sim N(0, t)$.

The expectation of A_t is:

$$E(A_t) = A_0 exp(\mu_A t)$$

At time T, the value of the equity will be:

$$E_T = max[A_T - D, 0]$$

The above shows that the value of the equity looks like the payoff of a (European) call option written on the value of the assets (A) with a strike price equal to the face value of the debt (D). Using Black-Scholes:

$$E_0 = A_0 N(d_1) - De^{-rT} N(d_2)$$

with

$$d_1 = \frac{ln\left(\frac{A_0 e^{rT}}{D}\right)}{\sigma_A \sqrt{T}} + \frac{1}{2}\sigma_A \sqrt{T}$$

$$d_2 = d_1 - \sigma_A \sqrt{T}$$

where r is the risk-free rate.

Let L be a measure of the leverage used by the company and defined as:

$$L = \frac{current\ value\ of\ debt}{current\ value\ of\ assets} = \frac{D\ e^{-rT}}{A_0}$$

Then we can write the current value of the Equity as:

$$E_0 = A_0[N(d_1) - L\ N(d_2)]$$

where,

$$d_1 = \frac{-\ ln(L)}{\sigma_A\sqrt{T}} + \frac{1}{2}\sigma_A\sqrt{T}$$

$$d_2 = d_1 - \sigma_A\sqrt{T}.$$

The current value of the debt (at time zero) is equal to:

$$B_0 = A_0 - E_0$$

Substituting for E_0 from above:

$$B_0 = A_0[N(-d_1) + L\ N(d_2)]$$

Note that the current value of debt B_0 can also be expressed by discounting the face value at the implied yield to maturity (y):

$$B_0 = De^{-yT} = D^{-rT}e^{(r-y)T} = A_0Le^{(r-y)T}$$

It follows that:

$$A_0Le^{(r-y)T} = A_0[N(-d_1) + LN(d_2)]$$

Therefore, the implied yield to maturity (y) can be calculated as:

$$y = r - \frac{ln\left(N(d_2) + \frac{N(-d_1)}{L}\right)}{T}$$

Then the implied credit spread (s_m)is calculated as:

$$s_m = y - r = -\frac{ln\left(N(d_2) + \frac{N(-d_1)}{L}\right)}{T}$$